11-4 Circles in the Coordinate Plane

TEKS FOCUS

TEKS (12)(E) Show that the equation of a circle with center at the origin and radius r is $x^2 + y^2 = r^2$ and determine the equation for the graph of a circle with radius r and center (h, k), $(x - h)^2 + (y - k)^2 = r^2$.

TEKS (1)(D) Communicate mathematical ideas, reasoning, and their **implications** using multiple **representations**, including symbols, diagrams, graphs, and language as appropriate.

Additional TEKS (1)(G), (2)(B)

VOCABULARY

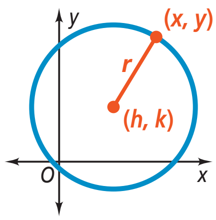
- **Standard form of the equation of a circle** – The standard form of an equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.
- **Implication** – a conclusion that follows from previously stated ideas or reasoning without being explicitly stated
- **Representation** – a way to display or describe information. You can use a representation to present mathematical ideas and data.

ESSENTIAL UNDERSTANDING

The information in the equation of a circle allows you to graph the circle.
Also, you can write the equation of a circle if you know its center and radius.

take note — Key Concept Equation of a Circle

An equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.



This is the **standard form of the equation of a circle**.

Problem 1

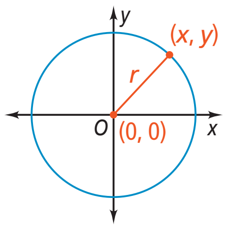
Think

What do you need to know to write the equation of any circle whose center is (0, 0)? You need to know the length of the radius or the coordinates of any point on the circle.

Deriving the Equation of a Circle Centered at the Origin

What is the standard form of an equation of a circle with center (0, 0)?

Use the Distance Formula to find an equation of a circle with center (0, 0) and radius r. Let (x, y) be any point on the circle. Then the radius r is the distance from (0, 0) to (x, y).



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$r = \sqrt{(x - 0)^2 + (y - 0)^2}$	Substitute r for d, (0, 0) for (x_1, y_1), and (x, y) for (x_2, y_2).
$r = \sqrt{x^2 + y^2}$	Simplify.
$r^2 = x^2 + y^2$	Square both sides.

The equation of a circle with radius r and center (0, 0) is $x^2 + y^2 = r^2$.

Problem 2

TEKS Process Standard (1)(G)

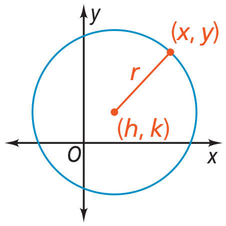
Think

How is this different from Problem 1? You still find the distance between the center of the circle and a point on the circle. The difference is the center is (h, k) instead of (0, 0).

Deriving the Equation of a Circle Centered at (*h*, *k*)

What is the standard form of an equation of a circle with center (h, k)?

Use the Distance Formula to find an equation of a circle with center (h, k) and radius r. Let (x, y) be any point on the circle. Then the radius r is the distance from (h, k) to (x, y).



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$r = \sqrt{(x - h)^2 + (y - k)^2}$	Substitute r for d, (x, y) for (x_2, y_2), and (h, k) for (x_1, y_1).
$r^2 = (x - h)^2 + (y - k)^2$	Square both sides.

The equation of a circle with radius r and center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$.

Problem 3

Plan

What do you need to know to write the equation of a circle? You need to know the values of h, k, and r; h is the x-coordinate of the center, k is the y-coordinate of the center, and r is the radius.

Writing the Equation of a Circle

What is the standard equation of the circle with center (5, −2) and radius 7?

$(x - h)^2 + (y - k)^2 = r^2$	Use the standard form of an equation of a circle.
$(x - 5)^2 + [y - (-2)]^2 = 7^2$	Substitute (5, −2) for (h, k) and 7 for r.
$(x - 5)^2 + (y + 2)^2 = 49$	Simplify.

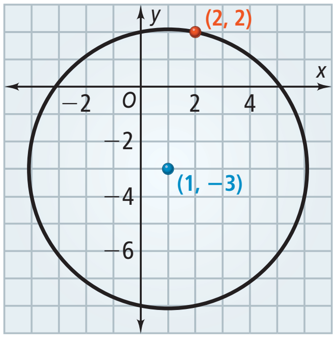
Think

How is this problem different from Problem 3?
In this problem, you don't know r. So the first step is to find r.

Problem 4

Using the Center and a Point on a Circle

What is the standard equation of the circle with center $(1, -3)$ that passes through the point $(2, 2)$?



Step 1 Use the Distance Formula to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Use the Distance Formula.

$= \sqrt{(1 - 2)^2 + (-3 - 2)^2}$ Substitute $(1, -3)$ for (x_2, y_2) and $(2, 2)$ for (x_1, y_1).

$= \sqrt{(-1)^2 + (-5)^2}$ Simplify.

$= \sqrt{26}$

Step 2 Use the radius and the center to write an equation.

$(x - h)^2 + (y - k)^2 = r^2$ Use the standard form of an equation of a circle.

$(x - 1)^2 + [y - (-3)]^2 = (\sqrt{26})^2$ Substitute $(1, -3)$ for (h, k) and $\sqrt{26}$ for r.

$(x - 1)^2 + (y + 3)^2 = 26$ Simplify.

Problem 5

TEKS Process Standard (1)(D)

Graphing a Circle Given Its Equation STEM

Communications **When you make a call on a cell phone, a tower receives and transmits the call. A way to monitor the range of a cell tower system is to use equations of circles. Suppose the equation $(x - 7)^2 + (y + 2)^2 = 64$ represents the position and the transmission range of a cell tower. What is the graph that shows the position and range of the tower?**

Know
The equation representing the cell tower's position and range

Need
To draw a graph

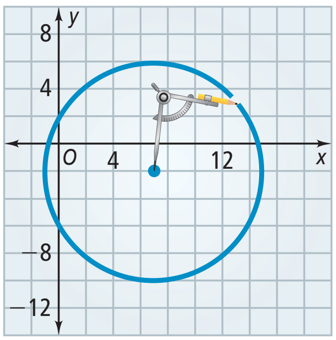
Plan
Determine the values of (h, k) and r in the equation. Then draw a graph.

$(x - 7)^2 + (y + 2)^2 = 64$ Use the standard equation of a circle.

$(x - \underset{\substack{\uparrow \\ h}}{7})^2 + [y - (\underset{\substack{\uparrow \\ k}}{-2})]^2 = \underset{\substack{\uparrow \\ r}}{8}^2$ Rewrite to find h, k, and r.

The center is $(7, -2)$ and the radius is 8.

To graph the circle, place the compass point at the center $(7, -2)$ and draw a circle with radius 8.



PRACTICE and APPLICATION EXERCISES

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For additional support when completing your homework, go to PearsonTEXAS.com.

Write the standard equation of each circle.

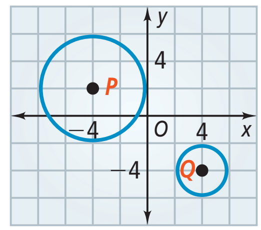
1. center $(2, -8)$; $r = 9$ **2.** center $(0, 3)$; $r = 7$ **3.** center $(0.2, 1.1)$; $r = 0.4$

4. center $(0, 0)$; $r = 4$ **5.** center $(-6, 3)$; $r = 8$ **6.** center $(-9, -4)$; $r = \sqrt{5}$

Find the center and radius of each circle. Then graph the circle.

7. $(x + 7)^2 + (y - 5)^2 = 16$ **8.** $(x - 3)^2 + (y + 8)^2 = 100$

Write a standard equation for each circle in the diagram at the right.



9. $\odot P$ **10.** $\odot Q$

Write the standard equation of the circle with the given center that passes through the given point.

11. center $(-2, 6)$; point $(-2, 10)$ **12.** center $(1, 2)$; point $(0, 6)$

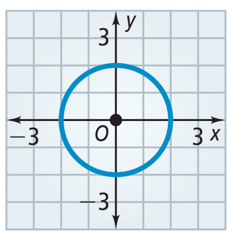
13. center $(7, -2)$; point $(1, -6)$ **14.** center $(-10, -5)$; point $(-5, 5)$

Apply Mathematics (1)(A) **Each equation models the position and range of a tornado alert siren. Describe the position and range of each.**

15. $(x - 5)^2 + (y - 7)^2 = 81$ **16.** $(x + 4)^2 + (y - 9)^2 = 144$

Write the standard equation of each circle.

17.



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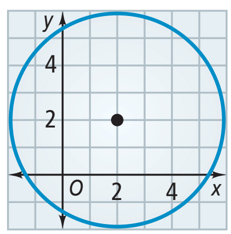
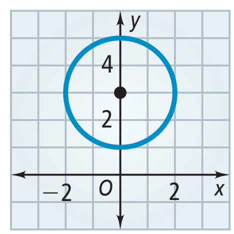


19.



Write an equation of a circle with diameter $\overline{AB}$.

20. $A(0, 0)$, $B(8, 6)$ **21.** $A(3, 0)$, $B(7, 6)$ **22.** $A(1, 1)$, $B(5, 5)$

Determine whether each equation is the equation of a circle. Justify your answer.

23. $x + (y - 3)^2 = 9$ **24.** $x + y = 9$ **25.** $(x - 1)^2 + (y + 2)^2 = 9$

26. **Analyze Mathematical Relationships (1)(F)** Find the circumference and area of the circle whose equation is $(x - 9)^2 + (y - 3)^2 = 64$. Leave your answers in terms of π.

27. **Explain Mathematical Ideas (1)(G)** Describe the graph of $x^2 + y^2 = r^2$ when $r = 0$.

28. The equations $(x + 6)^2 + (y + 5)^2 = 9$ and $(x + 6)^2 + (y + 5)^2 = 81$ represent two circles. Describe the relationship of the graphs.

29. The point $(2, 3)$ lies on a circle whose center is $(6, -1)$. What is the radius of the circle?

Sketch the graphs of each equation. Find all points of intersection of each pair of graphs.

30. $x^2 + y^2 = 13$
$y = -x + 5$

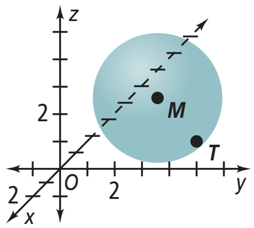
31. $x^2 + y^2 = 8$
$y = 2$

32. $(x - 2)^2 + (y - 2)^2 = 10$
$y = -\frac{1}{3}x + 6$

33. **Justify Mathematical Arguments (1)(G)** Derive the equation of a circle centered at (0, 0). Use the Distance Formula.

34. The concentric circles $(x - 3)^2 + (y - 5)^2 = 64$ and $(x - 3)^2 + (y - 5)^2 = 25$ form a ring. The lines $y = \frac{2}{3}x + 3$ and $y = 5$ intersect the ring, making four sections. Find the area of each section. Round your answers to the nearest tenth of a square unit.

35. **Use Multiple Representations to Communicate Mathematical Ideas (1)(D)** The equation of a sphere is similar to the equation of a circle. The equation of a sphere with center (h, j, k) and radius r is $(x - h)^2 + (y - j)^2 + (z - k)^2 = r^2$. In the diagram at the right, $M(-1, 3, 2)$ is the center of a sphere passing through the point T such that the radius of the sphere is $\sqrt{6}$. What is the equation of the sphere?



36. **Apply Mathematics (1)(A)** A close estimate of the radius of Earth's equator is 3960 mi.

a. Write the equation of the equator with the center of Earth as the origin.

b. Find the length of a 1° arc on the equator to the nearest tenth of a mile.

c. Columbus planned his trip to the East by going west. He thought each 1° arc was 45 mi long. He estimated that the trip would take 21 days. Use your answer to part (b) to find a better estimate.

TEXAS Test Practice

37. What is an equation of a circle with radius 16 and center $(2, -5)$?

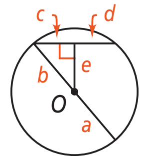
A. $(x - 2)^2 + (y + 5)^2 = 16$

B. $(x + 2)^2 + (y - 5)^2 = 4$

C. $(x + 2)^2 + (y - 5)^2 = 256$

D. $(x - 2)^2 + (y + 5)^2 = 256$

38. What can you NOT conclude from the diagram at the right?



F. $e < a$

G. $c^2 + e^2 = b^2$

H. $a = b$

J. $e = d$

39. Are the following statements equivalent?

- In a circle, if two central angles are congruent, then they have congruent arcs.
- In a circle, if two arcs are congruent, then they have congruent central angles.